Name _____

Interpretation of Confidence Interval vs. Confidence Level

Example: Suppose a random sample of 100 students from this school were surveyed if they own a computer. Forty-seven responded with "yes." A 95% confidence interval was calculated to be (0.37218, 0.56782).

C☺RRECT interpretation of Confidence Intervals

- ☺ We can be 95% confident that between 37% and 57% of all students at this school own computers.
- ☺ Based on my sample, I am 95% confident that the true population proportion of students at this school who own a computer falls between 37 and 57%.
- ☺ We are 95% confident that the population proportion of students at this school who own a computer is between 0.37 and 0.57.
- ☺ We are 95% sure that between 37% and 57% of all students at this school own a computer.
- ☺ We are 95% certain that the interval, 0.37 to 0.57, captures the true population proportion of students at this school who own a computer.

INC☹RRECT interpretation of Confidence Intervals

- ☹ 95% of students at this school own a computer.
- ☹ There is a 95% probability that the population proportion of students at this school owning a computer is between 0.37 and 0.57.
- ☹ We are 95% confident that the actual sample proportion of students at this school owning a computer is between 37% and 57%.
- ☹ We are 95% confident that between 37% and 57% of these 100 students at this school own a computer.
- ☹ In the long run, between 37% and 57% of students at this school will probably own a computer.

C☺RRECT interpretation of Confidence Level

- ☺ We have used a method to produce estimate intervals that is 95% of the time successful in capturing the actual population proportion of students at this school owning a computer.
- ☺ In repeated random sampling, approximately 95% of the constructed confidence intervals would contain the true population proportion of students at this school owning a computer.

INC☹RRECT interpretation of Confidence Level

- ☹ 95% is our confidence level.
- ☹ I am 95% confident in the level used when constructing my confidence interval.
- ☹ 95% of all confidence intervals of students owning a computer will fall between 37% and 57%.

Practice

1. If the 90% confidence interval of the proportion of a population is given by 0.45 ± 0.32, which of the following is correct?

a) There is a 90% probability that the true proportion is in the interval.
b) There is a 90% probability that the sample proportion is in the interval.
c) If 1,000 random samples of the same size are taken from the population, then approximately 900 of confidence intervals constructed will contain the true population proportion.
d) There is a 90% probability that a data value, chosen at random, will fall in this interval.
e) None of these is correct.

2. If the 95% confidence interval of the proportion of a population is 0.35 ± 0.025, which of the following is NOT correct?

a) If the sample size were to increase the width of the interval would decrease.
b) An increase in confidence level results in an increase in the width of the confidence interval.
c) This confidence interval could have been calculated after either a sample or a census was conducted.
d) If one would like a smaller confidence interval, one could increase sample size or decrease the confidence level.
e) All of these are correct.

3. The critical value for a 99.7% confidence interval for $\boldsymbol{p}$ is:

a) 1
b) 1.96
c) 2
d) 2.78
e) 3

4. Which of the following is NOT true about constructing confidence intervals?

a) The value of the standard error is a function of the sample statistics.
b) The center of the confidence interval is the population parameter.
c) One of the values that affects the width of a confidence interval is the sample size.
d) If the value of the population parameter is known, it is irrelevant to calculate a confidence interval for it.
e) The value of the level of confidence will affect the width of a confidence interval.

5. The confidence that we feel about a 90% confidence interval comes from the fact that

a) there is a 90% chance that the population parameter is contained in the confidence interval.
b) there is a 90% chance that the sample statistic is contained in the confidence interval.
c) 90% of confidence intervals constructed around a sample statistic will contain the population parameter.
d) the terms of confidence and probability are interchangeable.
e) the concepts of confidence and probability are synonymous.

Confidence Intervals Interpretation:

- 95% is not a probability. The probability is not 95% that a parameter is in the interval. The parameter is our target—it does not move. We shoot at the target by forming this interval—like throwing a horseshoe at a stake we cannot see. Sometimes (usually), we catch the stake somewhere within the horseshoe. Sometimes we miss. The probability that we found the stake is 100% when the horseshoe spans it and 0% when it does not. We just do not know which, so we prefer to say we have 95% confidence that we've captured the parameter. The 95% reflects our own uncertainty about the interval.
- Some people think that other sample results should fall in our interval. Different samples produce different intervals; 95% of those intervals contain the actual proportion/mean we wish we knew. Maybe ours is one of those intervals. Or maybe it's one of the other 5% that miss the target. We do not know. We never will. The other horseshoes need not land near ours---they are aiming at the stake, not at our horseshoe.
- Some people will think they are 95% sure they know the sample proportion/mean. Actually, we are sure of that. $\hat{p}$ and $\bar{x}$ are not just in the interval, it is (by construction) smack dab in the very center of the interval. Sample results are known, but populations are not. We are trying to catch a glimpse of the unknown, and that is what we will write conclusions about. We care about the stake, not the horseshoe.

Level of CI is related to margin of error, interval width, and sample size.

- It is common sense that very precise statements are risky. That is why the weatherman predicts we will get between 4 and 8 inches of snow. A prediction of 5 to 6 inches would be more helpful, but it is far less reliable.
- Kit is common sense that the more slack you cut yourself, the more faith you can have that you are right. We are very sure that between 1% and 50% of all cars are red.
- It is common sense that estimates based on larger samples are usually better. And better means that you get higher precision in your estimate (a narrower interval) or higher confidence, or both.

When determining sample size necessary to achieve a desired level of accuracy and confidence, the following two points are important:

- These calculations are actually part of the process of designing a study. We did not know how to do it when we discussed design issues in Chapter 5.
- We did say back then that the size of the sample needed depended on the level of accuracy desired, not on the size of the population. We see that the calculation rests only on level of confidence and margin of error, not on the size of the population. The calculation finds the minimal sample size that will work.