Honors Geometry 14.6 Notes

Name:_____ Date:_____ Period:_____

Sectors and Arcs

If a sector is a certain fraction of a circle, then its area is the same fraction of the circle's area. If an arc is a certain fraction of a circle, then its length is the same fraction of the circle's circumference.

Area (circle)

$$A = \pi r^2$$

Area of sectors of circle

(Sectors are similar to "pizza pie slices" of a circle.)

Semi-circle

(half of circle = half of area)

$$A = \frac{1}{2}\pi r^2$$

Quarter-Circle

(1/4 of circle = 1/4 of area)

$$A = \frac{1}{4}\pi r^2$$

Any Sector

(fractional part of the area)

$$A = \frac{n}{360}\pi r^2$$

where *n* is the number of degrees in the central angle of the sector.

Notice that when finding the area of a sector, you are actually finding a fractional part of the area of the entire circle. The fraction is determined by the ratio of the central angle of the sector to the "entire central angle" of 360 degrees, or by the ratio of the arc length to the entire circumference.

EXAMPLE:

Find the area of a sector with a central angle of 60 degrees and a radius of 10. Express answer to the *nearest tenth.*

Solution:

$$A = \frac{n}{360}\pi r^2$$

$$A = \frac{60}{360}\pi(10)^2$$

$$A = 52.35987756$$

$$A = 52.4$$

EXAMPLE:

Find the length of an arc with a central angle of 60 degrees and a radius of 10. Express answer to the *nearest tenth*

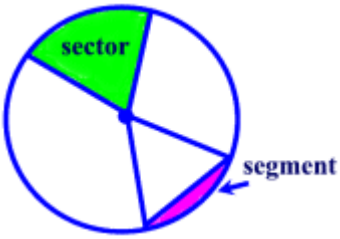
Solution:

$$Length = \frac{m}{360}2\pi r$$

$$Length = \frac{60}{360}2\pi(10)$$

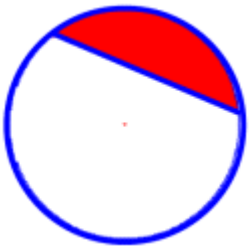
$$Length = 10.46666666$$

$$Length = 10.5$$



Segment of a Circle

While a sector looks like a "pie" slice, a **segment** looks like the "pie" slice with the triangular portion cut off. The segment is only the small partially curved figure left when the triangle is removed.



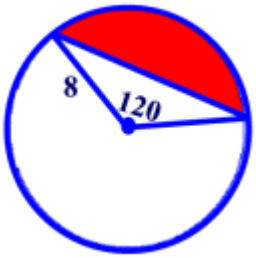
Definition: The **segment** of a circle is the region bounded by a chord and the arc subtended by the chord.

Finding the Area of a Segment of a Circle:

Dealing with the area of a segment is very similar to working with the area of a sector. If you find the area of the sector and subtract the area of the triangle, you will have the area of the segment portion of the circle. Let's look at an example problem.

Problem:

Find the area of a segment of a circle with a central angle of 120 degrees and a radius of 8 Express answer to *nearest integer.*

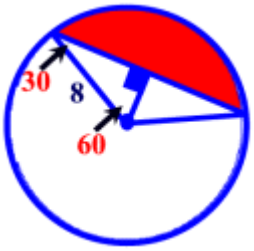


Solution:

Start by finding the area of the sector:

$$A = \frac{120}{360}\pi 8^2 = \frac{1}{3}\pi(64) = \frac{64}{3}\pi = 67.02064328$$

Now, find the area of the triangle. Dropping the altitude forms a 30-60-90 degree triangle. Using trig. (or the 30-60-90 rules), find the altitude, which is 4, and the other leg, which is 6.92820323 (or $4\sqrt{3}$).



$$A = \frac{1}{2}bh = \frac{1}{2}(2)(6.92820323)(4) = 27.71281292$$

$$A_{segment} = A_{sector} - A_{triangle}$$
$$A_{segment} = 67.02064328 - 27.71281292$$
$$A_{segment} = 39.30783036 = 39 \text{ square units}$$

$$A_{segment} = A_{sector} - A_{triangle}$$

In circle O with radius 12, the m∠COD = 30°. Find the length of arc CD and the area of sector COD.

Arc Length = _____ Area of Sector = _____

In circle O with diameter 20, m∠AOB = 72°. Find the length of arc AB and the area of sector AOB.

Arc Length = _____ Area of Sector = _____

In circle O with diameter 30, m∠AOB = 108°. Find the length of arc AB and the area of sector AOB.

Arc Length = _____ Area of Sector = _____

Find the area of the shaded region.