Calculus-y Things

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Calculus

- **Leibniz** and **Newton** independently invented calculus at the same-ish time
  - 17th century
- Derivatives = slope
- Integrals = area

LIMITS

- *The value a function approaches at a certain point*
- If results indeterminately, **L'Hopital's Rule** can be used
    - (will be explained in a later slide)
- Can be solved using **squeeze theorem** in certain cases
    - Aka pinch theorem, and sandwich theorem
    - (also will be explained later)

Derivatives

- Gives the *slope of a function*
  - Instantaneous rate of change
- Can be used to find max and min of a function
  - Occurs when derivative equals 0
- Power rule:
  - $y = x^n$
  - $y' = n*x^{(n-1)}$

y’ (“y prime”) or f’(x) or dy/dx are all ways to signify something is a derivative

Example: the derivative of x^3+2x is $3x^2+2$

INTEGRALS

- Also called “anti-derivative”
- Gives the *area of a function*
  - Finite
    - Subtract the integrals at the start and end points
  - Infinite
- Power rule:
  - $y = x^n$
  - $\int x^n dx = x^{n+1}/(n+1)$

This is just the notation

Example: the integral of x^3+2x is $(x^4/4)+x^2$

and the area between x=1 and x=2 is (16/4 + 4)-(¼+1) = 6.75

L'HOPITAL'S RULE

Indeterminate means you get n/0 where n is some number

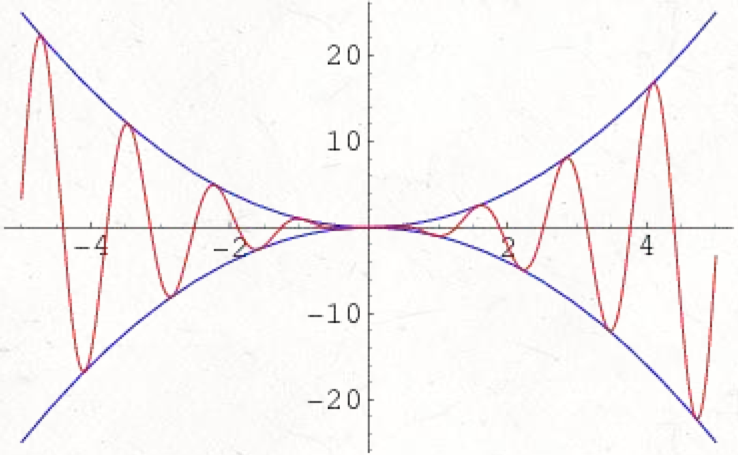
If a limit is indeterminate:

1. Take the derivative of the top function and the bottom function
2. Try the limit again

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Squeeze theorem

If you know a function is in between two other functions on some part of the graph, the limit of that function will be in between those functions



Example:

lim x→0 x^2*sin(1/x)

x^2*sin(1/x) is between $-x^2$ and x^2

At x=0, $-x^2$ and x^2 both equal 0

So lim x→0 x^2*sin(1/x) = 0

Toss ups

Question: Clairaut's theorem states that one form of this operation is commutative, and taking the dot product of a unit vector and a function's gradient yields the directional type of this operation. **This operation can be applied on a quotient to evaluate limits with indeterminate forms in L'Hôpital's rule.**

On any interval, there is at least one point where this operation is equal to the average slope of a function over that interval, according to the **mean-value theorem**.

Performing this operation on a composition of functions requires the **chain rule**.

For 10 points, name this operation from calculus that **finds a function's instantaneous rate of change,**

the inverse of integration

ANSWER**:** <u>differentiation</u> [accept word forms; or taking the derivative; accept partial differentiation or taking a partial derivative until "quotient"]

Question: This word describes all the points found in the closure of a set, including accumulation points. The "superior" and "inferior" forms of this operation are equal only for convergent sequences. The definition of "big-O" notation in asymptotic analysis uses one form of this operation. Weierstrass developed the first formal method of evaluating this operation, using "epsilons" and "deltas". **When dealing with indeterminate forms, l'Hôpital's rule is used to evaluate this operation**.

Applying this operation to difference quotients is a common way to define derivatives.

For 10 points, name this operation that yields **the value a function approaches** as its input **approaches a given value.**

ANSWER: limits [accept limit points]

Question: This action can be performed in a different coordinate system by using the Jacobian determinant. This operation over a region or its boundary are related by Stokes' theorem. Near vertical asymptotes, limits have to be used for the "improper" versions of them. You can do this to all rational functions by decomposing them into (*) partial fractions. It is usually defined as the limit of Riemann sums, which **approximate the area under a curve.**

For 10 points, name this operation that, by the fundamental theorem of calculus, yields the **antiderivative**.

ANSWER: <u>integration</u> [accept word forms and equivalents, like any answer that describes taking an integral]